

# Algebra 2

[Implement Start Year (2013-2014)]

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## Unit #6, Exponential/Logarithmic Functions

### Stage 1 – Desired Results

#### Established Goals

**2009 NJCCC Standard(s), Strand(s)/CPI #**  
(<http://www.nj.gov/education/cccs/2009/final.htm>)

**Common Core Curriculum Standards for Math and English**  
(<http://www.corestandards.org/>)

#### **Seeing Structure in Expressions A-SSE: 3c**

- Write expressions in equivalent forms to solve problems.

#### **Interpreting Functions F-IF: 7, 7e, 8b**

- Analyze functions using different representations

#### **Building Functions F-BF: 1b, 1c, 3, 5**

- Write a function that describes a relationship between two quantities.
- Build new functions from existing functions.

#### **Linear, Quadratic, and Exponential Models F-LE: 4**

- Construct and compare linear, quadratic, and exponential models and solve problems

#### 21<sup>st</sup> Century Themes

( [www.21stcenturyskills.org](http://www.21stcenturyskills.org) )

- Global Awareness
- Financial, Economic, Business and Entrepreneurial Literacy
- Civic Literacy
- Health Literacy
- Environmental Literacy

#### 21<sup>st</sup> Century Skills

*Learning and Innovation Skills:*

- Creativity and Innovation
- Critical Thinking and Problem Solving
- Communication and Collaboration

*Information, Media and Technology Skills:*

- Information Literacy
- Media Literacy
- ICT (Information, Communications and Technology) Literacy

	<p><i>Life and Career Skills:</i></p> <ul style="list-style-type: none"> <li><input checked="" type="checkbox"/> Flexibility and Adaptability</li> <li><input checked="" type="checkbox"/> Initiative and Self-Direction</li> <li><input checked="" type="checkbox"/> Social and Cross-Cultural Skills</li> <li><input checked="" type="checkbox"/> Productivity and Accountability</li> <li><input checked="" type="checkbox"/> Leadership and Responsibility</li> </ul>
<p><b>Enduring Understandings:</b> <i>Students will understand that . . .</i></p> <p>EU1</p> <ul style="list-style-type: none"> <li>• Graphs provide a means of seeing correlation of a function.</li> <li>• Graphs can be examined in detail or in their entirety depending on what information is sought.</li> </ul> <p>EU2</p> <ul style="list-style-type: none"> <li>• Inverses of functions provide information to solve problems.</li> </ul> <p>EU3</p> <ul style="list-style-type: none"> <li>• Symbolic statements can be manipulated to produce other statements of the same relationship.</li> </ul> <p>EU4</p> <ul style="list-style-type: none"> <li>• The characteristics of exponential and logarithmic functions and their representations are useful when solving real world problems.</li> </ul>	<p><b>Essential Questions:</b></p> <p>EU1</p> <ul style="list-style-type: none"> <li>• What is the correlation between the graphical and algebraic representations of an exponential function?</li> <li>• What is the correlation between the graphical and algebraic representations of a logarithmic function?</li> <li>• How do you model a quantity that changes regularly over time by the same percentage?</li> </ul> <p>EU2</p> <ul style="list-style-type: none"> <li>• How are exponents and logarithms related?</li> </ul> <p>EU3</p> <ul style="list-style-type: none"> <li>• Why is the form a function takes dependent on the purpose for which it is used?</li> <li>• When is it beneficial to have an equation in exponential form?</li> </ul> <p>EU4</p> <ul style="list-style-type: none"> <li>• What real world phenomena can be represented with mathematical functions?</li> <li>• How can you determine and use the relationships between models to investigate future situations?</li> <li>• What do the solutions to exponential functions provide?</li> <li>• What do the solutions to logarithmic functions provide?</li> <li>• What do the asymptotes tell about the exponential and logarithmic functions?</li> </ul>

<b>Knowledge:</b> <i>Students will know . . .</i>	<b>Skills:</b> <i>Students will be able to . . .</i>
<p>EU 1</p> <ul style="list-style-type: none"> <li>key features of the graphs of exponential and logarithmic functions (domain, range, intercepts, increasing/decreasing intervals, asymptotes)</li> <li>transformations for exponential and logarithmic functions</li> </ul> <p>EU 2</p> <ul style="list-style-type: none"> <li>logarithmic functions are the inverse of the exponential functions.</li> </ul> <p>EU3</p> <ul style="list-style-type: none"> <li>exponential rules are used to simplify exponential and logarithmic expressions.</li> <li>rewrite exponential and logarithmic expressions using exponent rules</li> </ul> <p>EU4</p> <ul style="list-style-type: none"> <li>evaluate exponential and logarithmic expressions and equations</li> <li>exponential and logarithmic functions and their equations can be used to solve real world problems.</li> <li>problems can be modeled using exponential or logarithmic functions.</li> </ul>	<p>EU 1</p> <ul style="list-style-type: none"> <li>graph exponential and logarithmic functions, with and without transformation.</li> <li>identify the domain and range of each function.</li> <li>identify the intercepts of the function.</li> <li>identify the intervals that the function is increasing and the intervals it's decreasing.</li> <li>identify where the asymptotes are and provide them in the graph of the function.</li> </ul> <p>EU 2</p> <ul style="list-style-type: none"> <li>find the inverse of the logarithmic function</li> <li>find the inverse of the exponential function</li> </ul> <p>EU 3</p> <ul style="list-style-type: none"> <li>condense logarithmic expressions and equations</li> <li>expand logarithmic expressions and equations</li> </ul> <p>EU4</p> <ul style="list-style-type: none"> <li>properties of logarithms to evaluate logarithmic expressions and equations</li> <li>properties of exponents to evaluate exponential expressions and equations</li> <li>translate and solve exponential functions to model real-world phenomena</li> </ul>

## Stage 2 – Assessment Evidence

Recommended Performance Task 1: EU1, EU4

### **A Graduation Present**

SURPRISE! Your grandparents offer you money for a down payment for a car as a graduation gift! However, you will receive the gift only if you agree to invest the money for at least 4 years. At that time, you hope to purchase a new car as a college graduation present to yourself. You hope to have \$7500 to make a down payment.

1. What amount, should you ask for AND at what interest rate, compounded monthly, would you need to invest your money so that you have at least \$7,500 accumulated in 4 years?
2. If you invest your money at a 5% interest rate compounded daily, how long would it take you to accumulate \$7,500?
3. Research 3 alternative investments and compare to the credit Which investment firm would you chose and why?

4. You know your grandfather will see the need for more money more clearly if you present the information in a table or graph. However, your grandmother, a retired math teacher, will expect an algebraic explanation.

**Recommended Performance Task 2: EU1, EU4**

**An iPad Present**

You want to convince your parents to buy you Apple's new generation iPad. However, your parents argue that Apple releases new generations so frequently, it would be a massive waste of money to buy the latest model. In fact, most gadgets depreciate exponentially. BUT WAIT! Your parents have decided to make a deal with you. If you can mathematically prove that the new iPad will be worth at least \$200 in five years, they will buy you one.

You are to write a report to present to your parents to prove the new iPad will be worth at least \$200 in five years. In your report please include the following:

- I. Research – cost of original iPad and cost of latest generation iPad
- II. Calculations
  - a. Exponential Model
  - b. Value after 5 years
  - c. Show work
- III. Graph of exponential model
- IV. Essay
  - a. Summary of methods and process(what you did in your calculations)
  - b. Explanation of why you want the latest generation iPad(how you are going to use it)

**Other Recommended Evidence:**

- Quiz on Graphing Exponential Functions
- Quiz on Graphing Logarithmic Functions
- Quiz on Condensing/Expanding Logarithmic Expressions
- Quiz on Solving Exponential and Logarithmic Equations
- Unit Test on Exponential and Logarithmic Functions

## Stage 3 – Learning Plan

**Suggested Learning Activities to Include Differentiated Instruction and Interdisciplinary Connections:** Consider the *WHERE TO* elements. Each learning activity listed must be accompanied by a learning goal of A= Acquiring basic knowledge and skills, M= Making meaning and/or a T= Transfer.

- Activity #1 – Hook- Doubling and “Halving” a number – (A)
- Activity #2 – Hook – NCAA Basketball Tournament – (M)
- Activity #3 – Nspire Activity – Graphing Logarithmic Functions - (M)  
<http://education.ti.com/calculators/timathnspired/US/Activities/Detail?sa=1010&t=1168&id=16057>
- Activity #4 – Graphing Calculator – Determining whether data best fits an exponential model – (M)
- Activity #5 – Research – Find application of Exponential growth and decay – (T)
- Activity #6 – Concentration Game – On index cards write 10 Exponential expressions and on 10 other write the Logarithmic equivalent – (A)
- Activity #7 – Simon Says – Have students demonstrate graphs of various functions using their hands – (T)

The following is the suggested sequence of learning activities and number of days for the Algebra 2 L2 class. Adjustments should be made accordingly for other levels.

### Approximately 15 days for completion of unit

YWBAT graph exponential growth and decay functions (A)

- HOOK – Activity #1 (A)
- Homework – Activity #2 (M)

YWBAT graph logarithmic equations (A)

- Activity #3 (M)

YWBAT condense and expand Logarithmic Functions (A)

YWBAT simplify and evaluate exponential and logarithmic expressions using inverse properties and properties of logarithms (A)

- Activity #4 (M)

YWBAT solve exponential and logarithmic equations using common logarithms and properties of logarithms (A)

- Activity #5 (T)

YWBAT evaluate expressions using base e and natural logarithm (A)

Activity #6 (A)

Activity #7 (T)

**Critical Vocabulary:**

Asymptote	Change of Base Formula	Common Logarithm	Continuously Compounded Interest
Exponential Decay	Exponential Growth	Exponential Equation	Exponential Function
Logarithm	Logarithmic Equation	Logarithmic Function	Natural Logarithm

**Activity 1**

Hook- Go around the class and have each student double the previous number, starting with the number 1. Write each number on the board. Note how quickly the numbers increase. Then start at 1 and have students give the fraction that is half the previous number. Write these alongside the doubled numbers. See if students recognize that these are reciprocals of the first set and note how quickly the numbers decrease. Tell them they will study growth and decay patterns in this lesson.

**Activity 2**

The NCAA holds a championship basketball tournament each spring. The nation's top 64 teams in Division 1 are invited to play. When a team loses, it is out of the tournament. Determine the number of teams left in the tournament after Round 5.

**Activity 4**

Graphing Calculator

**Exponential Functions in the Real World**

The United States has continued to grow since the late 1700's. The following chart shows how the number of people per square mile changed from 1790-1980.

Year	People Per Square Mile	Year	People Per Square Mile	Year	People Per Square Mile	Year	People Per Square Mile
1790	4.5	1840	9.8	1890	17.8	1940	37.2
1800	6.1	1850	7.9	1900	21.5	1950	42.6
1810	4.3	1860	10.6	1910	26.0	1960	50.6
1820	5.5	1870	10.9	1920	29.9	1970	57.5
1830	7.4	1880	14.2	1930	34.7	1980	64

1) Graph the following data on your calculator. (Remember, enter the numbers in the list, then go back to the Menu and use Stat to graph). What is happening in the graph?

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2) What is the line of best fit? Draw it as well to see if it works. \_\_\_\_\_

3) a) Using the equation from Question 2, what would be the number of people per square mile in 2000? Go online and find out the actual amount of people per square mile in the United States. How close is the prediction?

Prediction: \_\_\_\_\_ Actual: \_\_\_\_\_ Difference: \_\_\_\_\_

b) Was the prediction equation a good estimator? Why or why not?

\_\_\_\_\_

4) As stated in the title, this is an exponential function. What other type of function would create a similar outcome? Why?

\_\_\_\_\_

#### **Activity 5**

Have students describe two specific situations, one for exponential growth and one for exponential decay. For each, they should produce a function and a graph, and use them to generate a meaningful value.