Solving Systems of Linear Inequalities

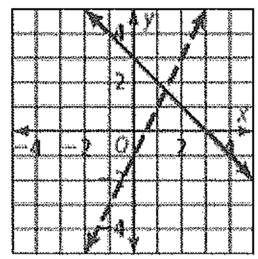
Problem

What is the solution of the system of inequalities? $\begin{cases} 2x - y > 1 \\ x + y \geq 3 \end{cases}$

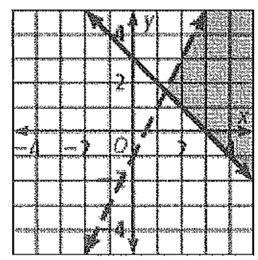
Step 1 Solve each inequality for y.

$$\begin{aligned} 2x - y &> 1 \\ -y &> -2x + 1 \\ y &< 2x - 1 \end{aligned} \qquad \text{and} \qquad \begin{aligned} x + y &\geq 3 \\ y &\geq -x + 3 \end{aligned}$$

Step 2 Graph the boundary lines. Use a solid line for $\geq$ or $\leq$ inequalities. Use a dotted line for $>$ and $<$ inequalities.



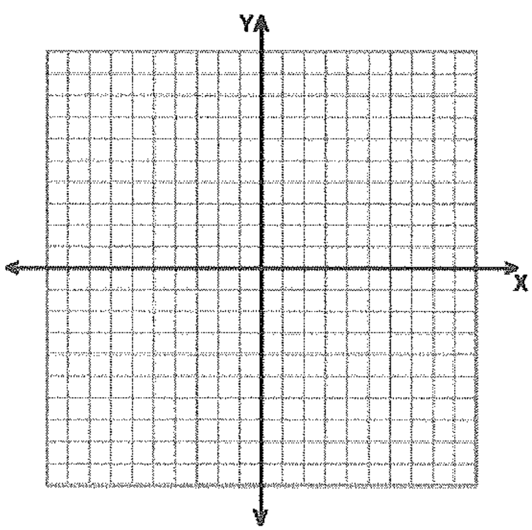
Step 3 Shade on the appropriate side of each boundary line. The overlap is the solution to the system.



Solve each system of inequalities by graphing.

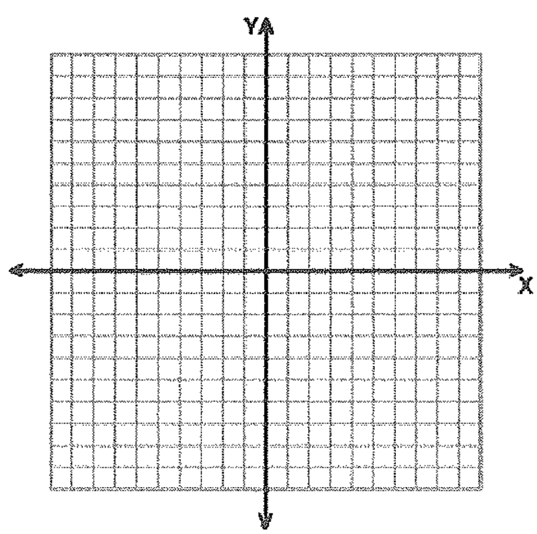
65.

$\begin{cases} y \leq x \\ y \geq 3x - 1 \end{cases}$



66.

$\begin{cases} y < -\frac{1}{3}x - 1 \\ y \geq 3x + 1 \end{cases}$



Properties of Exponents

VOCABULARY

The following are properties of exponents.

Let a and b be real numbers and let m and n be integers.

Product of Powers Property	$a^m \cdot a^n = a^{m+n}$
Power of a Power Property	$(a^m)^n = a^{mn}$
Power of a Product Property	$(ab)^m = a^m b^m$
Negative Exponent Property	$a^{-m} = \frac{1}{a^m}, a \neq 0$
Zero Exponent Property	$a^0 = 1, a \neq 0$
Quotient of Powers Property	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$
Power of a Quotient Property	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$

EXAMPLE ***Simplifying Algebraic Expressions***

Remember that simplified algebraic expressions contain only positive exponents.

a. $\frac{x^7y^4}{x^{-1}y^{-2}} = x^{7-(-1)}y^{4-(-2)}$ Quotient of powers property

$= x^8y^6$ Simplify exponents.

b. $(3x^{-2}y^4)^2 = 3^2(x^{-2})^2(y^4)^2$ Power of a product property

$= 9x^{-4}y^8$ Power of a power property

$= \frac{9y^8}{x^4}$ Negative exponent property

c. $(-2x^5y^3)^{-2} = (-2)^{-2}(x^5)^{-2}(y^3)^{-2}$ Power of a product property

$= (-2)^{-2}x^{-10}y^{-6}$ Power of a power property

$= \frac{1}{4x^{10}y^6}$ Negative exponent property

67. $x^3 \cdot x^2$	68. $\frac{2y^3}{y^5}$

69. $(3x)^2$	70. $\left(\frac{y}{2}\right)^3$
71. $(4x^3)^4$	72. x^0y^{-2}
73. $\frac{5x^2y}{2x^{-1}y^3}$	74. $\frac{-3xy}{9x^3y^{-4}}$
75. $\frac{-2x^2}{3xy^3} \cdot \frac{2x^{-1}}{y^{-1}}$	76. $\left(\frac{x^2y^5z}{2x^3}\right)^2$

Adding, Subtracting and Multiplying Polynomials

EXAMPLE 1 *Adding Polynomials Horizontally and Vertically*

Add the polynomials.

a. $(2x^2 + 8x + 4) + (x^2 - 8x - 2) = 2x^2 + x^2 + 8x - 8x + 4 - 2$

$= 3x^2 + 2$

b.

$$\begin{array}{r} 2x^3 - 3x^2 + x - 3 \\ + \quad\quad -x^2 + 2x + 4 \\ \hline 2x^3 - 4x^2 + 3x + 1 \end{array}$$

EXAMPLE 2 *Subtracting Polynomials Horizontally and Vertically*

a. $(3x^3 - 2x^2 + x) - (x^2 + 2x - 3) = 3x^3 - 2x^2 + x - x^2 - 2x + 3$ Add the opposite.

$= 3x^3 - 3x^2 - x + 3$ Add the opposite.

b.

$$\begin{array}{r} 4x^2 + x - 3 \\ -\ (2x^2 - 5x + 1) \\ \hline \end{array} \qquad \begin{array}{r} 4x^2 + x - 3 \\ -\ 2x^2 + 5x - 1 \\ \hline 2x^2 + 6x - 4 \end{array}$$

Find the sum or difference.

77. $(5x^2 + 2x + 1) + (4x^2 + 3x - 8)$	78. $(5x^2 - 6x - 1) - (4x^2 - 2x + 1)$
79. $(4x^2 + x + 6) + (7x - 10)$	80. $(x^2 - 2x + 7) - (-5x^2 - 3)$

EXAMPLE 3 *Multiplying Polynomials Horizontally and Vertically*

Find the product of the polynomials.

a. $(x-3)(x^2+3x+2) = (x-3)x^2 + (x-3)3x + (x-3)2$

$= x^3 - 3x^2 + 3x^2 - 9x + 2x - 6$

$= x^3 - 7x - 6$

b.

$3x^2 - 2x + 1$	
$\times \quad x + 2$	
$6x^2 - 4x + 2$	Multiply $3x^2 - 2x + 1$ by 2.
$3x^3 - 2x^2 + x$	Mulitply $3x^2 - 2x + 1$ by x.
$3x^3 + 4x^2 - 3x + 2$	Combine like terms.

Find the product.

81. $3x(x^2+x-2)$	82. $-2x(1-x-x^2)$
83. $(x-2)(x^2+2x+4)$	84. $(2x+1)(x^2-x-3)$
85. $(x-3)^2$	86. $(x-1)^3$

Factoring Quadratic Expressions

Problem

What is $6x^2 - 5x - 4$ in factored form?

$a = 6, b = -5$, and $c = -4$ — Find a, b, and c; they are the coefficients of each term.

$ac = -24$ and $b = -5$ — We are looking for factors with product ac and sum b.

Factors of −24	1, −24	−1, −24	2, −12	−2, 12	3, −8	−3, 8	4, −6	−4, 6
Sum of factors	−23	23	−10	10	−5	5	−2	2

The factors 3 and −8 are the combination whose sum is −5.

$6x^2 + 3x - 8x - 4$ — Rewrite the middle term using the factors you found.

$3x(2x + 1) - 4(2x + 1)$ — Find common factors by grouping the terms in pairs.

$(3x - 4)(2x + 1)$ — Rewrite using the Distributive Property.

Check $(3x - 4)(2x + 1)$ You can check your answer by multiplying the factors together.

$6x^2 + 3x - 8x - 4$

$6x^2 - 5x - 4$

Remember that not all quadratic expressions are factorable.

Factor each expression.

87. $x^2 + 6x + 8$	88. $x^2 - 4x + 3$
89. $2x^2 - 6x + 4$	90. $2x^2 - 11x + 5$
91. $4x^2 + 16x + 15$	92. $7x^2 - 19x - 6$

93. $x^2 - x - 72$	94. $2x^2 + 9x + 7$
95. $x^2 + 12x + 32$	96. $4x^2 - 28x + 49$

• $a^2 + 2ab + b^2 = (a+b)^2$ $a^2 - 2ab + b^2 = (a-b)^2$	Factoring perfect square trinomials
• $a^2 - b^2 = (a+b)(a-b)$	Factoring a difference of two squares

Problem

What is $25x^2 - 20x + 4$ in factored form?

There are three terms. Therefore, the expression may be a perfect square trinomial.

$a^2 = 25x^2$ and $b^2 = 4$ — Find a^2 and b^2.

$a = 5x$ and $b = 2$ — Take square roots to find a and b.

Check that the choice of a and b gives the correct middle term.

$2ab = 2 \cdot 5x \cdot 2 = 20x$

Write the factored form.

$a^2 - 2ab + b^2 = (a-b)^2$

$25x^2 - 20x + 4 = (5x-2)^2$

Check

$(5x-2)^2$	You can check your answer by multiplying the factors together.
$(5x-2)(5x-2)$	Rewrite the square in expanded form.
$25x^2 - 10x - 10x + 4$	Distribute.
$25x^2 - 20x + 4$	Simplify.

Factor each expression.

97. $x^2 - 12x + 36$	98. $x^2 + 30x + 225$

99. $9x^2 - 12x + 4$	100. $x^2 - 64$
101. $9x^2 - 42x + 49$	102. $25x^2 - 1$

Simplifying Radicals

VOCABULARY

Product Property The square root of a product equals the product of the square roots of the factors.

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \text{ when } a \text{ and } b \text{ are positive numbers}$$

Quotient Property The square root of a quotient equals the quotient of the square roots of the numerator and denominator.

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \text{ when } a \text{ and } b \text{ are positive numbers}$$

An expression with radicals is in **simplest form** if the following are true:

- No perfect square factors other than 1 are in the radicand.
- No fractions are in the radicand.
- No radicals appear in the denominator of a fraction.

EXAMPLE 1 *Simplifying with the Product Property*

Simplify the expression $\sqrt{147}$.

SOLUTION

You can use the product property to simplify a radical by removing perfect square factors from the radicand.

$\sqrt{147} = \sqrt{49 \cdot 3}$	Factor using perfect square factor.
$= \sqrt{49} \cdot \sqrt{3}$	Use product property.
$= 7\sqrt{3}$	Simplify.

EXAMPLE 2 *Simplifying with the Quotient Property*

Simplify the expression $\frac{\sqrt{63}}{6}$.

SOLUTION

$\frac{\sqrt{63}}{6} = \frac{\sqrt{9 \cdot 7}}{6}$	Factor using perfect square factor.
$= \frac{3\sqrt{7}}{6}$	Remove perfect square factor.
$= \frac{\sqrt{7}}{2}$	Divide out common factors.

Simplify the expression.

103. $\sqrt{50}$	104. $\sqrt{20}$
105. $\sqrt{240}$	106. $\sqrt{300}$
107. $\sqrt{3} \cdot \sqrt{12}$	108. $\sqrt{45}$
109. $\sqrt{\frac{16}{25}}$	110. $\sqrt{\frac{1}{9}}$
111. $\sqrt{\frac{3}{16}}$	112. $5\sqrt{\frac{20}{49}}$
113. $\frac{\sqrt{40}}{14}$	114. $10\sqrt{\frac{20}{64}}$
115. $\sqrt{\frac{9}{81}}$	116. $\frac{1}{5}\sqrt{\frac{25}{64}}$