Real Numbers and Number Operations

VOCABULARY

The **graph** of a real number is the point on a real number line that corresponds to the number. On a number line, the numbers increase from left to right, and the point labeled 0 is the origin.

The number that corresponds to a point on a number line is the **coordinate** of the point.

The **opposite,** or *additive inverse,* of any number a is $-a$.

The **reciprocal,** or *multiplicative inverse,* of any nonzero number a is $\frac{1}{a}$.

EXAMPLE 1 *Graphing and Ordering Real Numbers*

Graph and write the numbers in increasing order: $-\sqrt{7}, 0, \frac{3}{2}, 2, \frac{3}{8}$.

SOLUTION

$-\sqrt{7} \approx -2.6, \frac{3}{2} = 1.5, \frac{3}{8} \approx 0.4$ — Rewrite each number in decimal form.

Plot the points on the real number line.

$-\sqrt{7}, 0, \frac{3}{8}, \frac{3}{2}, 2$ — Write the numbers from least to greatest.

Write the numbers in increasing order.

1. $1, \frac{1}{3}, \sqrt{2}$	2. $\sqrt{68}, 8, \sqrt{62}$
3. $\sqrt{5}, \frac{2}{3}, 3.25$	4. $-\sqrt{2}, -15, -5.7$

Algebraic Expressions

VOCABULARY

A **variable** is a letter that is used to represent one or more numbers.

An **algebraic expression** is an expression involving variables.

Like terms are expressions that have the same variable part. **Constant terms** such as -4 and 2 are also like terms.

The **base** of an exponent is the number or variable that is used as a factor in repeated multiplication. For example, in the expression 4^b, 4 is the base.

An **exponent** is the number or variable that represents the number of times the base is used as a factor. For example, in the expression 4^b, b is the exponent.

A **power** is the result of repeated multiplication. For example, in the expression $4^2 = 16$, 16 is the second power of 4.

Any number used to replace a variable is the **value of the variable.**

When the variables in an algebraic expression are replaced by numbers, the result is called the **value of the expression.**

Terms are the parts that are added in an expression, such as 5 and $-x$ in the expression $5 - x$.

A **coefficient** is the number multiplied by a variable in a term.

Two algebraic expressions are **equivalent** if they have the same value for all values of their variable(s).

EXAMPLE 1 ***Using Order of Operations***

$2(3 + 18 \div 3^2 - 7)$	$= 2(3 + 18 \div 9 - 7)$	Evaluate the power.
	$= 2(3 + 2 - 7)$	Divide.
	$= 2(-2)$	Add within parentheses.
	$= -4$	Multiply.

Evaluate the expression without using a calculator.

5. $(-1+3)-4^2$	6. $5-(-2+4)^2$
7. $36 \div (-3)^2 - 1$	8. -5^2

EXAMPLE 2 *Evaluating an Algebraic Expression*

Evaluate $2t^2 - 3$ when $t = 4$.

SOLUTION

$2t^2 - 3$	$= 2(4)^2 - 3$	Substitute 4 for t.
	$= 2(16) - 3$	Evaluate the power.
	$= 32 - 3$	Multiply.
	$= 29$	Subtract.

Evaluate the expression.

9. $x^2(4 - x)$ when x = 2	10. $x - (x + 5)$ when x = 20
11. $4x - 3y + 2$ when x = 4 and y = -3	12. $9(m - n)^2$ when m=4 and n=1

EXAMPLE 3 *Simplifying by Combining Like Terms*

Simplify $6(x - y) - 4(x - y)$.

SOLUTION

$6(x - y) - 4(x - y)$	$= 6x - 6y - 4x + 4y$	Distributive property
	$= (6x - 4x) + (-6y + 4y)$	Group like terms.
	$= 2x - 2y$	Combine like terms.

Simplify the expression.

13. $7x - (9x + 5)$	14. $2(n^2 + n) - 5(n^2 - 4n)$
15. $-6x^2 + 4x - x^2 + 15x$	16. $7x - 2y + 3 - 9y + 4 + 5x$

Solving Linear Equations

VOCABULARY

An **equation** is a statement in which two expressions are equal.

A **linear equation** in one variable is an expression that can be written in the form $ax = b$ where a and b are constants and $a \neq 0$.

A number is a **solution** of an equation if the statement is true when the number is substituted for the variable.

Two equations are **equivalent** if they have the same solutions.

EXAMPLE 1 *Variable on One Side*

Solve $-19 = -2y + 5$.

SOLUTION

$-19 = -2y + 5$	Write original equation.
$-24 = -2y$	To isolate y, subtract 5 from each side.
$12 = y$	Divide each side by -2.

EXAMPLE 2 *Variable on Both Sides*

Solve $4x - 2x = 15 - 3x$.

SOLUTION

$4x - 2x = 15 - 3x$	Write original equation.
$2x = 15 - 3x$	Combine like terms.
$5x = 15$	To collect the variable terms, add $3x$ to each side.
$x = 3$	Divide each side by 5.

EXAMPLE 3 *Using the Distributive Property*

Solve $15(4 - y) = 5(10 + 2y)$.

SOLUTION

$15(4 - y) = 5(10 + 2y)$	Write original equation.
$60 - 15y = 50 + 10y$	Distributive property
$60 = 50 + 25y$	To collect the variable terms, add $15y$ to each side.
$10 = 25y$	Subtract 50 from each side.
$\frac{2}{5} = y$	Divide each side by 25.

EXAMPLE 4 *Solving an Equation with Fractions*

Solve $\frac{2}{3}x + \frac{3}{5} = \frac{4}{15}$.

SOLUTION

$\frac{2}{3}x + \frac{3}{5} = \frac{4}{15}$	Write original equation.
$15\left(\frac{2}{3}x + \frac{3}{5}\right) = 15\left(\frac{4}{15}\right)$	Multiply each side by the LCD, 15.
$10x + 9 = 4$	Distributive property
$10x = -5$	To isolate x, subtract 9 from each side.
$x = -\frac{1}{2}$	Divide each side by 10.

Solve the equation.

17. $-18 = y + 6$	18. $15 - 3a = -4a + 16$
19. $5(x - 3) + 12 = -2(x - 2)$	20. $6n = \frac{2}{3}(5n - 2)$
21. $\frac{3}{5}x = \frac{2}{3}x + 1$	22. $10(x + 3) - (-9x - 4) = x - 5 + 3$
23. $-12(x - 12) = -9(1 + 7x)$	24. $-3(1 + 6r) = 14 - r$
25. $-4(k - 2) + 3(k + 1) = 7$	26. $3x - 9 = 2(x - 5)$

Solving Linear Inequalities

As with an equation, the solutions of an inequality are numbers that make it true. The procedure for solving a linear inequality is much like the one for solving linear equations. To isolate the variable on one side of the inequality, perform the same algebraic operation on each side of the inequality symbol.

The **Addition and Subtraction Properties of Inequality** state that adding or subtracting the same number from both sides of the inequality does not change the inequality.

If $a < b$, then $a + c < b + c$. If $a < b$, then $a - c < b - c$.

The **Multiplication and Division Properties of Inequality** state that multiplying or dividing both sides of the inequality by the same *positive* number does not change the inequality.

If $a < b$ and $c > 0$, then $ac < bc$. If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$.

Problem

What is the solution of $3(x + 2) - 5 \le 21 - x$? Graph the solution.

Justify each line in the solution by naming one of the properties of inequalities.

$3x + 6 - 5 \le 21 - x$	Distributive Property
$3x + 1 \le 21 - x$	Simplify.
$4x + 1 \le 21$	Addition Property of Inequality
$4x \le 20$	Subtraction Property of Inequality
$x \le 5$	Division Property of Inequality

To graph the solution, locate the boundary point. Plot a point at $x = 5$. Because the inequality is *"less than or equal to,"* the boundary point is part of the solution set. Therefore, use a closed dot to graph the boundary point. Shade the number line to the left of the boundary point because the inequality is *"less than."*

Graph the solution on a number line.

Solve each inequality. Graph the solution.

27. $2x + 4(x - 2) > 4$	28. $4 - (2x - 4) \ge 5 - (4x + 3)$

The procedure for solving an inequality is similar to the procedure for solving an equation but with one important exception.

The Multiplication and Division Properties of Equality also state that, when you multiply or divide each side of an inequality by a negative number, you must reverse the inequality symbol.

If $a < b$ and $c < 0$, then $ac > bc$. If $a < b$ and $c < 0$, then $\frac{a}{c} > \frac{b}{c}$.

Problem

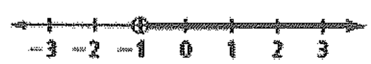
What is the solution of $2x - 3(x - 1) < x + 5$? Graph the solution.

Justify each line in the solution by naming one of the properties of inequalities.

$2x - 3(x - 1) < x + 5$	
$2x - 3x + 3 < x + 5$	Distributive Property
$-x + 3 < x + 5$	Simplify.
$-2x < 2$	Subtraction Property of Inequality
$x > -1$	Division Property of Inequality

The direction of the inequality changed in the last step because we divided both sides of the inequality by a negative number.

Graph the solution on a number line.



Solve each inequality. Graph the solution.

29. $x - 1 \leq -4(-2 - x)$

30. $7 - 7(x - 7) > -4 + 5x$

Relations and Functions

- A **relation** is a set of ordered pairs.
- The **domain** is the set of the first numbers in each pair, or the x-values.
- The **range** is the set of the second numbers in each pair, or the y-values.
- A relation is a **function** if each input value x corresponds to exactly one output value y. In a set of ordered pairs for a function, an x-value cannot be repeated with two or more different y-values.

Problem

Roll a number cube to find six ordered pairs. Determine whether the set of ordered pairs is a function. Find the domain and range.

Roll a number cube six times to get the x-values of the six ordered pairs. Roll it six more times to get the y-values of the ordered pairs.

{(6, 1), (2, 1), (5, 4), (2, 2), (1, 4), (4, 2)} Write the ordered pairs.

{(6, 1), (2, 1), (5, 4), (2, 2), (1, 4), (4, 2)} Circle any x-values that repeat with different y-values to determine whether the relation is a function.

The x-value 2 is repeated with two different y-values so the relation is not a function.

The domain is the set of first numbers in each pair: {1, 2, 4, 5, 6}.
The range is the set of second numbers in each pair: {1, 2, 4}.

Determine whether each relation is a function. Explain your answer. Find the domain and range of each relation.

31. {(1, 2), (1, 3), (1, 4), (1, 5), (1, 6)}	32. {(0, −1), (1, 2), (−1, −1), (−2, 5), (2, 9)}
33. {(A, B), (C, D), (E, F), (G, H)}	34. {(I, M), (N, P), (I, T), (I, P)}

You can write a rule for a function using function notation. Function notation makes it easier to identify the input and output for a particular function, and to compare two or more functions.

	Example	Independent Variable (input)	Dependent Variable (output)
Function Rule	$y = 3x - 5$	x	y
To write the function rule using function notation, replace *y* with *f(x)* (read *f(x)* as "*f* of *x*").	$f(x) = 3x - 5$	x	$f(x)$
The function, *f* evaluated at 2 or *f*(2).	$f(2) = 3(2) - 5$ $= 6 - 5$ $= 1$	2	1

Problem

What is the value of the function for the given value of x? Write the input, x, and the output, $f(x)$, as an ordered pair.

$f(x) = 4x - 2$ for $x = -3$

$f(-3) = 4(-3) - 2$ Replace x with -3.

$f(-3) = -14$ Simplify.

$(-3, -14)$ Write the input and output as an ordered pair.

Evaluate each function for the given value of x, and write the input and output as an ordered pair.

35. $f(x) = 3x - 7$ for $x = 6$	36. $g(x) = 9x - 5$ for $x = 3$
37. $h(x) = 12x$ for $x = 4$	38. $t(x) = 8x - 5$ for $x = 7$

Linear Functions and Slope Intercept Form

> You can use the slope-intercept form to write equations of lines.
>
> - The slope-intercept formula is $y = mx + b$, where m represents the slope of the line, and b represents its y-intercept. The y-intercept is the point at which the line crosses the y-axis.
> - The slope of a horizontal line is always zero, and the slope of a vertical line is always undefined.

Problem

What is the equation of the line that contains the point (3, −1) and has a slope of $-\frac{4}{3}$?

$-1 = \left(-\frac{4}{3}\right)(3) + b$	To find b, substitute the values $-\frac{4}{3}$ for m, 3 for x, and −1 for y into the slope-intercept formula.
$-1 = -4 + b$	Multiply.
$3 = b$	Add 4 to each side and simplify.
$y = -\frac{4}{3}x + 3$	Substitute $-\frac{4}{3}$ for m and 3 for b into the slope-intercept formula.

Write an equation for each line.

39. $m = 4$; contains (3, 2)	40. $m = -2$; contains (4, 7)
41. $m = 0$; contains (3, 0)	42. $m = 3$; contains (−2, −4)